Directed Reading Spring 2026

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Motivation: Representing Primes

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The Path Forward: To distinguish between these cases, we must move beyond specific examples and develop a general theory for:

$$f(x, y) = ax^2 + bxy + cy^2$$

What are Quadratic Forms?

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Example: $f(x, y) = x^2 + 2xy + 5y^2$

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$$f(2, 2) = 4 + 8 + 20 = 32$$

32 is not properly represented by $f(2, 2)$

Equivalent Forms, Reduction

Two forms $f(x, y), g(x, y)$ are equivalent if there exist integers p, q, r, s such that

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A primitive, positive definite form $ax^2 + bxy + cy^2$ is said to be reduced if

$$|b| \leq a \leq c, \text{ and } b \geq 0 \text{ if either } |b| = a, \text{ or } a = c$$

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Consider that this is equivalent to saying that

$$f\left(M\begin{bmatrix}x\\y\end{bmatrix}\right) \equiv g\begin{bmatrix}x\\y\end{bmatrix} \iff f(x + y, y) = g(x, y)$$

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where $M = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$
Since $\det M = ps - qr = 1 - 0 = 1$, these forms are properly equivalent, and M is orientation preserving.

Groups

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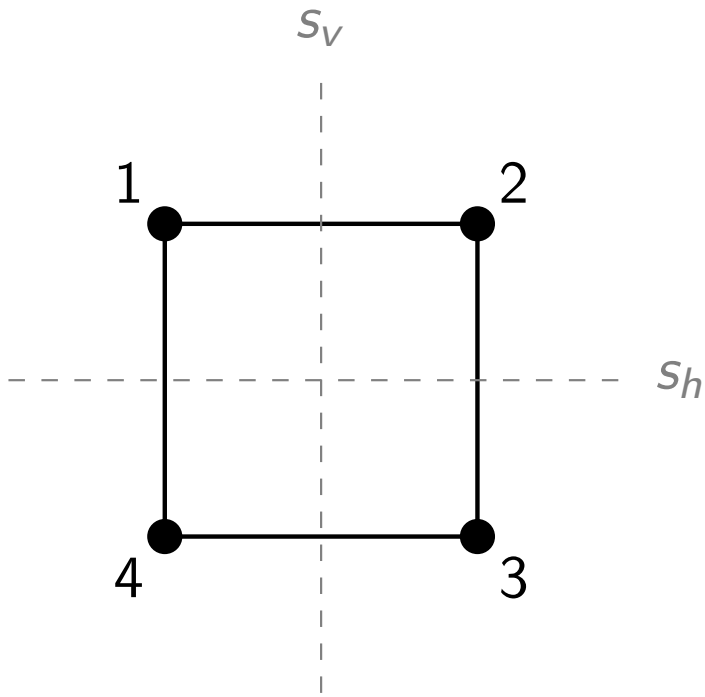
Importantly, a group induces a structure on some type of object such that combining things in the group preserves that structure.

Group Example: Symmetries of the Square (D_4)



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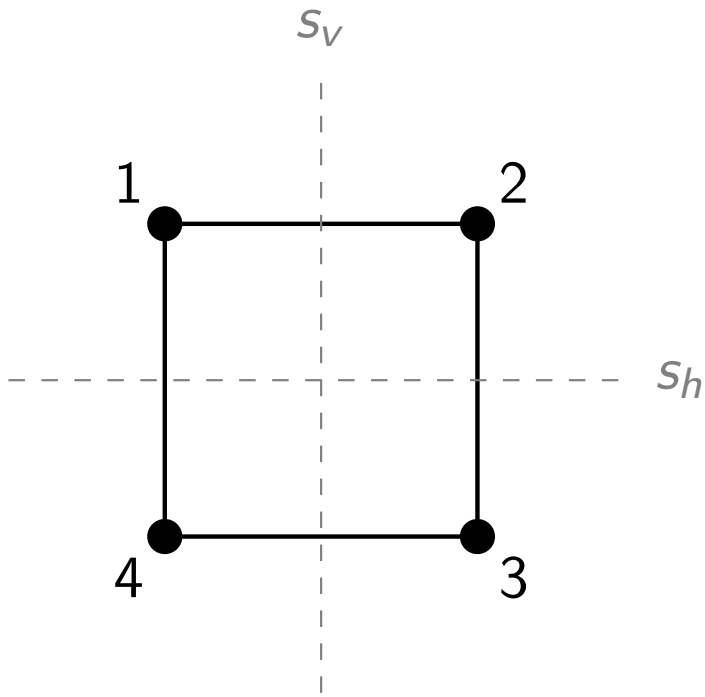

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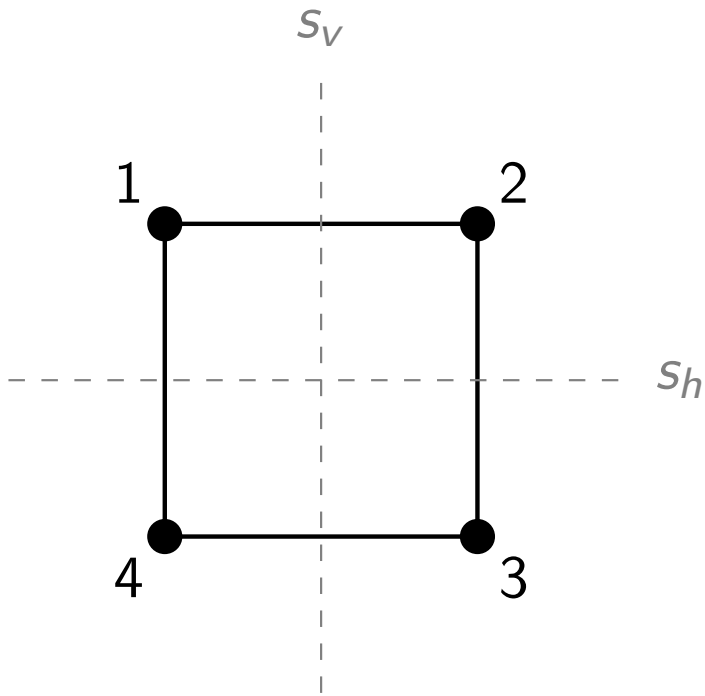

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- 4 Reflections: Horizontal, Vertical, and 2 Diagonals.

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where B is chosen so that

$$B \equiv b \pmod{2a}, \qquad B \equiv b' \pmod{2a'}, \qquad B^2 \equiv D \pmod{4aa'}.$$

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The composed form is

$$(fg)(x, y) = F(x, y) = 4x^2 + xy + 2y^2$$

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This gives the intuitive motivation for the notion of a class group.

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By finding a method to relate quadratic forms to a well studied field, *group theory*, we are able to better understand and work with these quadratic forms and primes.